Feedback Delay Network

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A feedback delay network, as described in audio signal processing literature, can be viewed as a generalized state space filter. These notes summarize my view of an FDN and provide some straightforward design equations.

1 State Space Representation of an FDN

Figure 1 illustrates a state space representation of a feedback delay network. It is identical to the standard state space representation of a linear, time-invariant system, except that the single sample period delay is replaced by a more complex transform. For our purposes, the transform $\mathbf{H}(z)$ represents a bank of delay lines representing propagation delay and frequency-dependent attenuation. Specifically

$$\mathbf{H}(z) = \begin{bmatrix} z^{-M_1}A_1(z) & 0 & \dots & 0 \\ 0 & z^{-M_2}A_2(z) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z^{-M_N}A_N(z) \end{bmatrix} \tag{1}$$

where

$$A_i(z) = \frac{g_i}{1 - d_i z^{-1}} \tag{2}$$

The time-domain relationship between the outputs x_i and the inputs v_i of the delay ban is

$$x_i(n) = d_i x_i(n-1) + g_i v_i(n - M_i) \tag{3}$$

The rest of the system is described by the following equations

$$\mathbf{v} = \mathbf{A}\mathbf{s} + \mathbf{B}x \tag{4}$$

$$y = \mathbf{C}\mathbf{s} \tag{5}$$

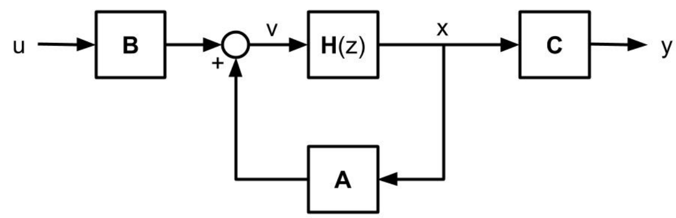

Figure 1: Feedback Delay Network Block Diagram

where

$$A_{ij} = \frac{1}{N} - \delta_{ij} \tag{6}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix}^T \tag{7}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix} / N \tag{8}$$

Our choices for **A**, **B**, and **C** lead to fairly straightforward results. The output is given by

$$y(n) = \frac{1}{N} \sum_{i=1}^{N} x_i(n) \tag{9}$$

The delay bank inputs are given by

$$\begin{aligned} v_i(n) &= \sum_{j=1}^{N} A_{ij} x_j(n) + u(n) \\ &= \sum_{j=1}^{N} \left(\frac{1}{N} - \delta_{ij} \right) x_j(n) + u(n) \\ &= y(n) + u(n) - x_i(n) \end{aligned} \tag{10}$$

This can also be written in matrix notation as

$$\mathbf{v}(n) = \mathbf{B}[y(n) + u(n)] - \mathbf{x}(n) \tag{11}$$

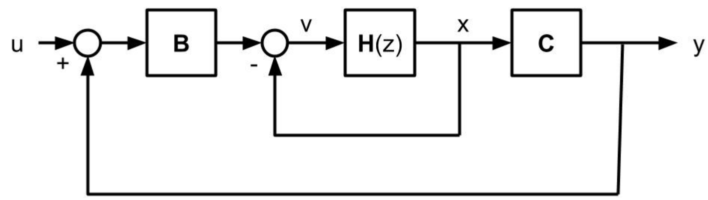

Figure 2: Simplified Feedback Delay Network

Figure 2 illustrates this simplified structure, which is valid for the values of **A**, **B**, and **C** that we have chosen.

2 Delay Bank Parameters

As stated above, the FDN includes N delay lines represented in the frequency domain as

$$h(z) = z^{-M} \frac{g}{1 - dz^{-1}} \tag{12}$$

This delay line represents a propagation path between two points in the space of interest. The sample delay z^{-M} accounts for the propagation delay and the single-pole filter accounts for loss due to absorption. For a path of length L, the sample delay M is given by

$$M = \text{round}(f_s L/c) \tag{13}$$

where f_s is the sampling rate in Hz and c is the speed of sound. The magnitude of the loss is given by

$$A(\Omega) = \frac{g}{|1 - de^{-j\Omega}|} \tag{14}$$

where Ω is the frequency in radians/sample. Since the absorption of sound by air increases significantly with frequency, we will assume the following:

$$A(0) = 1 \tag{15}$$

$$\begin{aligned} A(\pi) &= 10^{-\alpha L/20} \\ &= A_{\mathrm{HF}} \end{aligned} \tag{16}$$

where α is the absorption coefficient in dB/m at the Nyquist frequency. This leads to the following equations for g and d:

$$\begin{aligned} g &= \frac{2A_{\mathrm{HF}}}{1 + A_{\mathrm{HF}}} \\ d &= \frac{1 - A_{\mathrm{HF}}}{1 + A_{\mathrm{HF}}} \end{aligned} \tag{17}$$

Given the path length, high-frequency absorption coefficient, and the sampling rate, we can us Equations 17 and 13 to determine the parameters M, g, and d of the delay line. In order to complete the delay bank, we need to decide the number of delay lines N and a set of path lengths. In my experience, fewer than ten delay lines (say $N = 6$) are sufficient to produce a nice reverb effect. I use an exponential sequence of path lengths ranging from $L_1 = L_{\max}$ to $L_N = L_{\max}/10$. I determine $L_{\max}$ from the room area, which I presume to be given, based on some assumptions about the shape of the room.